Indexing Music Collections through Graph Spectra

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1 Graph Representation

- Pitch classes are mapped into graph nodes
- Intervals are mapped into graph arrows

\[ V_G = \{C, E, F, G, A\} \]
\[ A_G = \{(C, E), (E, G), (G, A), (A, F), (F, E), (E, C)\} \]

1.1 Example

An instance of the folk song “In Frankrijk buiten de poorten” (2nd version), OGL19306-2.

2 Indexing

Algebraic structures are associated to the graph:
- Adjacency matrix \( A(G) \)
- Laplacian matrix \( L(G) = D(G) - A(G) \)

The topology of a graph is encoded through its Laplacian spectrum \( \Lambda \).

\[ L(G) = P \Lambda P^T \text{ where } \Lambda = \text{diag} (\lambda_1, \lambda_2, \ldots, \lambda_{|V|}) \]

2.1 Computing similarity

To compute the similarity between two graphs \( G_1 \) and \( G_2 \), we compute the Euclidean distance between their signatures, which is inversely proportional to the structural similarity of the graphs.

\[ d(G_1, G_2) = \sqrt{\sum_{i=1}^{\mid V \mid} (\lambda_i(1,1) - \lambda_i(2,1))^2} \]

3 Experiments


- Experiments conducted on a subset of this resource, that consists of 141 songs, of which we used the first phrase.
- These songs have been classified in 18 classes or melody groups.

<table>
<thead>
<tr>
<th>CRITERIA</th>
<th>NN 1st tier</th>
<th>2nd tier</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAPLACIAN</td>
<td>66%</td>
<td>44%</td>
</tr>
<tr>
<td>ADJACENCY</td>
<td>58%</td>
<td>28%</td>
</tr>
<tr>
<td>Opti3</td>
<td>40%</td>
<td>39%</td>
</tr>
<tr>
<td>EMD</td>
<td>64%</td>
<td>33%</td>
</tr>
<tr>
<td>PTD</td>
<td>64%</td>
<td>30%</td>
</tr>
</tbody>
</table>

Nearest neighbour (NN), first tier and second tier results on the Onder de groene linde collection, computed using Laplacian spectra and Adjacency spectra of the graphs.

The results have been compared to the following melodic similarity measures:
- the optimal distance measure established by Müllensiefen and Frieler (Opti3)
- the Earth Mover’s Distance (EMD)
- the Proportional Transportation Distance (PTD)

3.1 Example of structurally similar songs

Example of songs with similar structure: “Heer Halewijn” (3rd version) OGL19205 with its NN, OGL19107, instance of “Heer Halewijn” (4th version).

Graph representation of the folk songs OGL19205 and OGL19107 (without the eigenloop on B) shown here above. The two letters in each circle represent the pitch classes respectively in the first and the second song.